Colloquium Interactions Mathématique et Informatique

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Apprentissage d'equilibres de Nash, de comportements collectifs efficaces. Dynamiques inertielles et Algorithmes de calcul parallele.

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1. SOLVING COMPLEX PROBLEMS: GOAL, SETTING

- “Model” general mechanisms (individual, collective, strategic aspects) used by human beings in order to solve efficiently real world problems (economic, social...).
- Complexity: cognitive, psychological, environment (unknown, changing), multiscales.
- H. Simon (Nobel prize of economics, 1978, artificial intelligence): as the complexity of the problem increases, the responses of the agents tend to be more and more simple.

Solving complex problems is a dynamical process:

- Agents try to improve step by step their performances.
- Performance = routine, way of doing, periodic orbit, maintenance, training.
- Procedural rationality: from heuristics, satisficing, to local and global optimization.
- Exploration, learning, adaptive aspects (Levinthal and Warglien, 1999), (Sobel, 2000), (Berthoz, 2003).
- Multiscaling: short, long memory (E. Kandel, Nobel prize of medecine, 2000).
- Inertia aspects: costs to quit or enter a routine, reactivity (physiological, psychological).

Worthwhile to move principle (Attouch and Soubeyran, 2005):
Acceptable transitions: Advantages to change are greater or equal than Costs to change.

SOLVING COMPLEX PROBLEMS (continuation)

- Model: *Inertial nonautonomous dynamical potential games*.

  Individual, collective, decentralized, local, inertia, adaptive, learning, multiscale aspects.

  Two types of models:

  1. Alternating games.
  2. Simultaneous games (a coordination process takes account of the collective aspects).

- Results: Convergence results to Nash equilibria, or Pareto (learning cooperative behavior), rate of convergence (finite time convergence), algorithms (alternate, parallel).

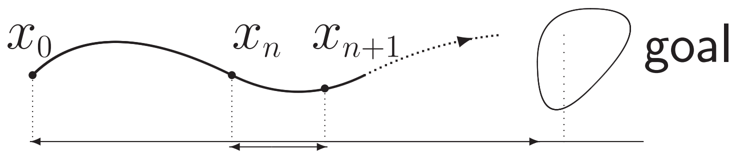
  Links with continuous dynamical systems, first and second order gradient systems on structured decision spaces (Hilbert, Riemannian, metric).

- Applications: From human sciences to computing and artificial intelligence.

  1. Analyze coordination and efficiency of human organizations, the learning, adaptive, inertial features and the dynamical approach to equilibria (Nash, Pareto,...), attractors.
  2. Numerical purpose (decomposition, splitting methods, parallel computing).
  3. Control theory, stabilization of coupled systems, automatic.
  4. Artificial intelligence, robotic. Transpose to multi-agents systems the above algorithms.

CONTENTS

2. DYNAMICS WITH INERTIA: WORTHWHILE TO MOVE PRINCIPLE



Worthwhile to move principle: Attouch and Soubeyran (2005).

$$\text{Structured decision space} \quad \begin{cases} X: \ \text{state, strategy , performance space} \\ g: X \longrightarrow \mathbb{R} \ \ \text{gain function} \\ c: X \times X \longrightarrow \mathbb{R}^+ \ \text{cost to change} \end{cases}$$

Routine = temporary exploration-exploitation phase.

Passing from a routine x_n to a next routine x_{n+1}: Advantages to move must be greater or equal than some fraction of Costs to move (not too much sacrificing):

$$g(x_{n+1}) - g(x_n) \ \geq \ \ \theta_n \ c(x_{n+1}, x_n)$$

$$\Updownarrow$$

Marginal gain $\geq$ Cost to change

DYNAMICS WITH INERTIA: PROXIMAL ALGORITHMS

Worthwhile to move principle + Optimization aspects ⇒ Proximal Dynamics.

Difficulty to change, inertia $\longrightarrow$ Anchoring, local effects:
At stage n and performance x_n, the net gain function of the agent is given by

$$\xi \longmapsto g(\xi) - c(\xi, x_n).$$

General proximal dynamics:

$$x_{n+1} \in \epsilon_n - \mathrm{argmax}\{g(\xi) - \theta_n c(\xi, x_n) \ : \ \xi \in E(x_n, r_n)\}$$

- ϵ_n : psychological, cognitive features (motivation, degree of resolution)
- θ_n : cognitive features (speed, learning, reactivity).
- $E(x_n, r_n)$: exploration set.

Classical prox. dynamics: H Hilbert, $c(\xi, x) = \| \xi - x \|^2$, $\lambda_n > 0$

$$x_{n+1} \in \mathrm{argmax}\{g(\xi) - \frac{1}{2\lambda_n} \| \xi - x_n \|^2 \ : \ \xi \in H\}$$

$$\boxed{x_{n+1} \ \in \ \epsilon_n - \text{argmax}\{g(\xi) - \theta_n c(\xi, x_n) \ : \ \xi \in E(x_n, r_n)\}}$$

Take $\xi = x_n$,

$$\epsilon_n + g(x_{n+1}) - g(x_n) \geq \theta_n c(x_{n+1}, x_n).$$

Sum w.r. to n, " *finite resource assumption*" $\sup_X g < +\infty$

$$\sum_n \theta_n c(x_{n+1}, x_n) \leq \sup_X g - g(x_0) + \sum_n \epsilon_n.$$

Suppose $\sum_n \epsilon_n < +\infty$ and $\theta_n > \underline{\theta} > 0$.

Then,

$$\sum_n c(x_{n+1}, x_n) < +\infty.$$

Hence the local features of the general proximal algorithms

$$c(x_{n+1}, x_n) \longrightarrow 0 \text{ as } \ n \rightarrow +\infty.$$

<u>Convergence results</u>:

- Concave case: Martinet, Rockafellar (1976), Auslender, Lemaire, Güler...
- Nonconcave case: Att-Bolte, Math programming, (2006).

3. POTENTIAL GAMES: THE STATIC NORMAL FORM

(Monderer-Shapley, 1996), m agents (players), indexed by $i \in \{1, 2, ..., m\}$.

- H_i = performance (strategy) space of player i, $x_i \in H_i$.
- $f_i : H_i \to [-\infty, +\infty[$, individual utility of player i. Without interaction with other agents, the goal of player i is to maximize his individual utility: $\text{maximize}_{x_i \in H_i} f_i(x_i)$.
- $\Psi : \otimes H_i \to \mathbb{R}$, collective utility function, models the collective welfare of the group.
- Static payoff function of player i:

$$\boxed{F_i(x_1, x_2, ..., x_m) = f_i(x_i) + \beta_i \Psi(x_1, x_2, ..., x_m)}$$

- Static Nash equilibria:

$$\boxed{x_i \in \text{argmax}_{\xi \in H_i} \{f_i(\xi) + \beta_i \Psi(x_1, x_2, ..., x_{i-1}, \xi, x_{i+1} ..., x_m)\}, i = 1, 2, ..., m.}$$

- Potential function: $L_\beta(x_1, x_2, ..., x_m) = \Psi(x_1, x_2, ..., x_m) + \sum_{i=1}^{m} \frac{1}{\beta_i} f_i(x_i)$.
  Best reply = It is "as if" each player maximizes the same objective:

$$\boxed{\text{maximize}_{x_1 \in H_1, .., x_m \in H_m} \{\Psi(x_1, x_2, ..., x_m) + \sum_{i=1}^{m} \frac{1}{\beta_i} f_i(x_i)\}.}$$

4.1: ALTERNATING POTENTIAL GAMES with INERTIA, 2 PLAYERS

Alternating projections on closed affine subspaces (Von Neumann, 1950, Annals of Math). Best reply alternating dynamic with cost to change, m=2, H_i Hilbert space, i = 1, 2:

$$\boldsymbol{x}_n = (x_{1,n}, x_{2,n}) \longrightarrow (x_{1,n+1}, x_{2,n}) \longrightarrow \boldsymbol{x}_{n+1} = (x_{1,n+1}, x_{2,n+1}) \quad n = 0, 1, \ldots$$

$$\boxed{\begin{cases} x_{1,n+1} \in \text{argmax}\{f_1(\xi) + \beta_1 \Psi(\xi, x_{2,n}) - \frac{\alpha}{2} \parallel \xi - x_{1,n} \parallel^2_{\mathcal{H}_1} : \xi \in \mathcal{H}_1\} \\ x_{2,n+1} \in \text{argmax}\{f_2(\eta) + \beta_2 \Psi(x_{1,n+1}, \eta) - \frac{\nu}{2} \parallel \eta - x_{2,n} \parallel^2_{\mathcal{H}_2} : \ \eta \in \mathcal{H}_2\} \end{cases}}$$

∞

- Concave case, Ψ quadratic semidefinite negative form:
  $(x_{1,n}, x_{2,n})$ converges weakly ($n \to +\infty$) to a Nash equilibrium = max. point of L_β.
  Att.-Redont-Soubeyran, SIAM J. Optim., 2007.
  Att.-Bolte-Redont-Soubeyran, JCA., 2008.

- Nonconcave case, finite dimension, the potential L_β satisfies the Kurdyka-Lojasiewicz inequality (analytic, nonsmooth semialgebric function...), smooth coupling function Ψ:
  $(x_{1,n}, x_{2,n})$ converges to a critical point of the potential L_β, finite length trajectory.
  Att.-Bolte-Redont-Soubeyran, HAL 2008, submitted MOR.

4.2: ALTERNATING POTENTIAL GAMES with INERTIA: m PLAYERS

- H_i = performance (strategy) space of player $i \in 1, 2, ..., m$ = real Hilbert space.
- $f_i : H_i \to [-\infty, +\infty[$, individual utility of player i, concave upper semicontinuous.
- $\Psi : \otimes H_i \to \mathbb{R}$, collective utility function: $\Psi(x_1, x_2, ..., x_m) = \sum_{1 \leq i < j \leq m} \Psi_{i,j}(x_i, x_j)$ where $\Psi_{i,j}$ is a semi-definite negative quadratic form.
- Potential function: $L_\beta(x_1, x_2, ..., x_m) = \Psi(x_1, x_2, ..., x_m) + \sum_{i=1}^{m} \frac{1}{\beta_i} f_i(x_i)$ has at least a maximum point.

$$\left\{ \begin{array}{l} x_{1,n+1} \in \text{argmax}\{f_1(\xi) + \beta_1 \Psi(\xi, x_{2,n}, \ldots, x_{m,n}) - \frac{1}{2} \parallel \xi - x_{1,n} \parallel^2 : \xi \in H_1\} \\ \vdots \\ x_{i,n+1} \in \text{argmax}\{f_i(\xi) + \beta_i \Psi(x_{1,n+1}, ..., x_{i-1,n+1}, \xi, x_{i+1,n}, \ldots, x_{m,n}) - \frac{1}{2} \parallel \xi - x_{i,n} \parallel^2\} \\ \vdots \\ x_{m,n+1} \in \text{argmax}\{f_m(\xi) + \beta_m \Psi(x_{1,n+1}, ..., x_{m-1,n+1}, \xi) - \frac{1}{2} \parallel \xi - x_{m,n} \parallel^2 : \ \xi \in H_m\} \end{array} \right.$$

Theorem (ABRS): *Let $\boldsymbol{x}_n = (x_{1,n}, ..., x_{m,n})$ a sequence generated by the algorithm.*

1. $\boldsymbol{x}_n$ *converges weakly to a maximum point* $\boldsymbol{x}_\infty$ of L_β.
2. $\boldsymbol{x}_n$ *is a maximizing sequence for* L_β, $f_i(x_{i,n}) \to f_i(x_{i,\infty})$, $\Psi(\boldsymbol{x}_n) \to \Psi(\boldsymbol{x}_\infty)$.

4.3: MULTISCALING: FROM NASH TO PARETO EQUILIBRIA

Best reply alternating dynamic with cost to change and multiscale aspects:

- m=2, H_i Hilbert space, i = 1, 2
- Ψ concave, bounded from above, $C =$ argmax$\Psi = \Psi^{-1}(0)$ non empty closed convex.

$$\boldsymbol{x}_n = (x_{1,n}, x_{2,n}) \longrightarrow (x_{1,n+1}, x_{2,n}) \longrightarrow \boldsymbol{x}_{n+1} = (x_{1,n+1}, x_{2,n+1}) \quad n = 0, 1, \ldots$$

$$\begin{cases} x_{1,n+1} \in \text{argmax}\{f_1(\xi) + \beta_{1,n}\Psi(\xi, x_{2,n}) - \frac{\alpha}{2} \parallel \xi - x_{1,n} \parallel^2_{\mathcal{H}_1} : \xi \in \mathcal{H}_1\} \\ x_{2,n+1} \in \text{argmax}\{f_2(\eta) + \beta_{2,n}\Psi(x_{1,n+1}, \eta) - \frac{\nu}{2} \parallel \eta - x_{2,n} \parallel^2_{\mathcal{H}_2} : \ \eta \in \mathcal{H}_2\} \end{cases}$$

- Fast growing of the collective learning parameters: $\beta_{1,n} = \beta_n \beta_1$, $\beta_{2,n} = \beta_n \beta_2$ with $\beta_n \to +\infty$ as $n \to +\infty$. Limiting equilibria:

$$\text{maximize}\{\frac{1}{\beta_1} f_1(x_1) + \frac{1}{\beta_2} f_2(x_2) : \Psi(x_1, x_2) = 0\}.$$

- Slow vanishing of the collective learning parameters: $\beta_{1,n} = \beta_n \beta_1$, $\beta_{2,n} = \beta_n \beta_2$ with $\beta_n \to 0$ as $n \to +\infty$. Limiting equilibria:

$$\text{maximize}\{\Psi(x_1, x_2) : x_1 \in \text{argmax} f_1, \ x_2 \in \text{argmax} f_2)\}.$$

5. SIMULTANEOUS COORDINATED GAMES with INERTIA

Att.-Bruceno-Combettes, working paper, 2008.

- m players indexed by $i \in \{1, 2, ..., m\}$, H_i Hilbert space.
- $f_i : H_i \longrightarrow \mathbb{R} \cup \{-\infty\}$ the *individual* payoff of player i coming from his own action (concave upper semicontinuous).
- The coupling term Ψ takes care of the *common* payoff. A decentralized structure involves *subgroups*. Within each subgroup, agents share similar features and interact actively.
- For each subgroup $k \in \{1, ..., p\}$ and player $i \in \{1, ..., m\}$, $L_{ki} : H_i \longrightarrow \mathcal{G}_k$ is a bounded linear operator, it measures the implication of player i in the subgroup k. The common payoff relative to the activity of the subgroup k:

$$\Psi_k(x_1, ..., x_m) = \varphi_k\left(\sum_{i=1}^m L_{ki} x_i \right)$$

where $\varphi_k : \mathcal{G}_k \mapsto \mathbb{R}$ is a concave continuous function.

- The global collective payoff which results from the joint action of all the players:

$$\boxed{\Psi(x_1, ..., x_m) = \sum_{k=1}^p \varphi_k\left(\sum_{i=1}^m L_{ki} x_i \right).}$$

DYNAMIC OF DECISION: $\boldsymbol{x}_n \mapsto \boldsymbol{x}_{n+1}$ where $\boldsymbol{x}_n = (x_{1,n}, \ldots, x_{m,n})$.

Collective phase: The task of the co-ordination process (for example a supervisor..., see H. Moulin, *Fair division and Collective Welfare*, MIT Press, 2003) is to evaluate the quality of the global collective payoff $\boldsymbol{x} \mapsto \Psi(\boldsymbol{x})$ around $\boldsymbol{x}_n$ and to determine the best possible direction making this global performance increase, namely $\nabla\Psi(\boldsymbol{x}_n)$.

Following a gradient rule, the co-ordinator proposes (possibly imposes), to each player i, to modify his current stategy $x_{i,n}$:

$$x_{i,n} \mapsto \xi_{i,n} = x_{i,n} + \gamma_n \nabla_i \Psi(\boldsymbol{x}_n).$$

Equivalently,

$$\xi_{i,n} = x_{i,n} + \gamma_n \left(\sum_{k=1}^{p} L_{ki}^* \nabla\varphi_k \left(\sum_{j=1}^{m} L_{kj} x_{j,n} \right)\right).$$

γ_n is a parameter which takes account of various features of the decision process:

- Learning collective behaviors (speed, reactivity).
- Interaction agent-coordinator: It is part of the skill of the co-ordinator to help to determine the size of the coefficient γ_n: not too small in order to improve substantially the performance, but not too large, otherwise it is out of the capabilities of the agent, and out of the local character of the gradient rules.

Individual phase:

- Each player i has to find a new optimal strategy $x_{i,n+1}$ taking account both of his individual payoff $f_i(.)$ and of the collective constraints (not to be far from the prescribed modified strategy $\xi_{i,n}$), which leads to the following maximization problem:

$$\boxed{x_{i,n+1} \in \operatorname{argmax}\{f_i(\xi) - \frac{1}{2\gamma_n} \parallel \xi - \xi_{i,n} \parallel^2_{\mathcal{H}_i} : \ \xi \in \mathcal{H}_i\}.}$$

Equivalently,

$$\boxed{x_{i,n+1} \in \operatorname{argmax}\{f_i(\xi) - \frac{1}{2\gamma_n} \parallel \xi - (x_{i,n} + \gamma_n \nabla_i \Psi(\boldsymbol{x}_n)) \parallel^2_{\mathcal{H}_i} : \ \xi \in \mathcal{H}_i\}.}$$

$$\text{where} \quad \nabla_i \Psi(\boldsymbol{x}_n) = \textstyle\sum_{k=1}^{p} L^*_{ki} \nabla \varphi_k \Big(\textstyle\sum_{j=1}^{m} L_{kj} x_{j,n} \Big).$$

- Parallel splitting algorithm (linked with simultaneous games).
- Risk aversion: relaxation parameter $\lambda_{i,n}$

$$x_{i,n+1} = \lambda_{i,n} x_{i,n} + (1 - \lambda_{i,n}) \operatorname{argmax}\{f_i(\xi) - \tfrac{1}{2\gamma_n} \parallel \xi - (x_{i,n} + \gamma_n \nabla_i \Psi(\boldsymbol{x}_n)) \parallel^2_{\mathcal{H}_i} : \xi \in \mathcal{H}_i\}.$$

Convergence results:

$$x_{i,n+1} = \lambda_n x_{i,n} + (1-\lambda_n)\,\text{argmax}\{f_i(\xi) - \frac{1}{2\gamma_n} \parallel \xi - (x_{i,n} + \gamma_n \nabla_i \Psi(\boldsymbol{x}_n)) \parallel^2_{\mathcal{H}_i} \colon \xi \in \mathcal{H}_i\}.$$

$$\nabla_i \Psi(\boldsymbol{x}_n) = \textstyle\sum_{k=1}^{p} L_{ki}^* \nabla \varphi_k \Big(\sum_{j=1}^{m} L_{kj} x_{j,n} \Big)$$

Assumptions:

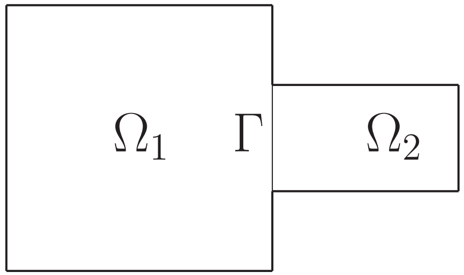
- φ_k is concave differentiable with a τ_k- Lipschitz continuous gradient, $\tau_k \in (0, +\infty)$.
- Set $\beta = \left(p \max\limits_{1 \le k \le p} \tau_k \sum\limits_{i=1}^{m} \|L_{ki}\|^2 \right)^{-1}$ and fix $\varepsilon \in \,]0, \min\{1, \beta\}[$.
- Suppose $\gamma_n \in [\varepsilon, 2\beta - \varepsilon]$, $\lambda_n \in [0, 1-\varepsilon]$.

Theorem (ABC) *Under the above assumptions, the sequence* $(\boldsymbol{x}_n)_{n\in\mathbb{N}}$ *converges weakly to a solution of the maximization problem*

$$\underset{x_1 \in \mathcal{H}_1, \ldots, x_m \in \mathcal{H}_m}{\text{maximize}} \quad \sum_{i=1}^{m} f_i(x_i) + \sum_{k=1}^{p} \varphi_k \Big(\sum_{j=1}^{m} L_{jk} x_j \Big)$$

which can be equivalently interpreted as a Nash equilibrium of the corresponding normal potential game.

DECOMPOSITION OF DOMAINS IN PDE's.



Dirichlet problem on Ω: $h \in L^2(\Omega)$ given, find $u : \Omega \longrightarrow \mathbb{R}$ solution of

$$\begin{cases} -\Delta u = h \quad \text{on } \Omega \\ u = 0 \quad \text{on } \partial\Omega \end{cases}$$

Variational formulation:

$$\min \left\{ \tfrac{1}{2} \int_{\Omega_1} \nabla v_1|^2 + \tfrac{1}{2} \int_{\Omega_2} |\nabla v_2|^2 - \int_\Omega hv : \quad v_1 \in X_1,\ v_2 \in X_2, \quad [v] = 0 \quad \text{on } \Gamma \right\}.$$

- $X_i = \{v \in \ H^1(\Omega_i), \quad v = 0$ on $\partial\Omega \cap \partial\Omega_i\}, \quad v = v_i$ on $\Omega_i,\ i = 1, 2.$
- $[v] =$ jump of v through the interface Γ.

$$\boxed{\begin{array}{c} \min \{f_1(v_1) + f_2(v_2) : v_1 \in X_1, v_2 \in X_2, \quad A_1(v_1) - A_2(v_2) = 0\}. \\ A_i : H^1(\Omega_i) \to \mathcal{Z} = L^2(\Gamma) \text{ is the trace operator}, i = 1, 2. \end{array}}$$

Continuous dynamical system:

$$\begin{cases} -\Delta \dfrac{\partial u_1}{\partial t} - \Delta u_1 = h_1 \ \text{ on } \Omega_1 \\ -\Delta \dfrac{\partial u_2}{\partial t} - \Delta u_2 = h_2 \ \text{ on } \Omega_2 \\ \dfrac{\partial \dot{u}_1(t)}{\partial \nu_1} + \dfrac{\partial u_1}{\partial \nu_1}(t) - \beta(t)\,[u(t)] = 0 \ \text{ on } \Gamma \\ \dfrac{\partial \dot{u}_2(t)}{\partial \nu_2} + \dfrac{\partial u_2}{\partial \nu_2}(t) + \beta(t)\,[u(t)] = 0 \ \text{ on } \Gamma \end{cases}$$

Discrete version: Alternating Algorithm with Dirichlet-Neumann transmission conditions:

$$(u_{1,n}, u_{2,n}) \longrightarrow (u_{1,n+1}, u_{2,n}) \longrightarrow (u_{1,n+1}, u_{2,n+1}) \text{ with } \beta_n \longrightarrow +\infty.$$

$$\begin{cases} -(1+\alpha)\Delta u_{1,n+1} = h_1 - \alpha \Delta u_{1,n} \ \text{ on } \Omega_1 \\ (1+\alpha)\dfrac{\partial u_{1,n+1}}{\partial \nu_1} + \beta_n u_{1,n+1} = \beta_n u_{2,n} + \alpha \dfrac{\partial u_{1,n}}{\partial \nu_1} \ \text{ on } \Gamma \\ u_{1,n+1} = 0 \ \text{ on } \partial\Omega_1 \cap \partial\Omega \end{cases}$$

$$\begin{cases} -(1+\alpha)\Delta u_{2,n+1} = h_2 - \alpha \Delta u_{2,n} \ \text{ on } \Omega_2 \\ (1+\alpha)\dfrac{\partial u_{2,n+1}}{\partial \nu_2} + \beta_n u_{2,n+1} = \beta_k u_{1,n+1} + \alpha \dfrac{\partial u_{2,n}}{\partial \nu_2} \ \text{ on } \Gamma \\ u_{2,n+1} = 0 \ \text{ on } \partial\Omega_2 \cap \partial\Omega \end{cases}$$

PERSPECTIVES

- Multiscale aspects: different speed of learning (agents, groups, individual and collective).
- Multi-agents, IA., general model with adaptive aspects (payoff, learning...), control.
- Learning curves, routines.
- Probabilistic aspects: payoff as expectation.
- From gradient methods to Riemannian gradient methods, interior point methods.
- From learning coefficients to learning operators.
- Second order dynamics, reactivity aspects.
- Zero sum games, conflictual aspects, hybrid dynamics (continuous and discrete).
- From convex to non-convex analysis via Kurdyka-Lojasiewicz inequality in simultaneous games.
- Numerical developments for domain decomposition problems.

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